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Elastic Decohesive Rheology in CICE

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Sandia National Labs

Collaborators: Elizabeth Hunke (LANL),
Deborah Sulsky (UNM), Howard Schreyer (UNM)

CESM PCWG Meeting
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- Sea ice models generally use variations of viscous-plastic rheology - Hibler (1979) *J. Phys. Oceanography*
  - assumes that cracks are always present
  - strength depends on thickness and fractional area
  - isotropic weakening

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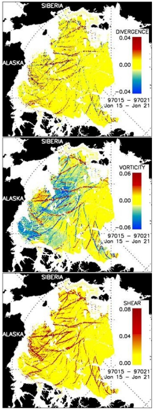

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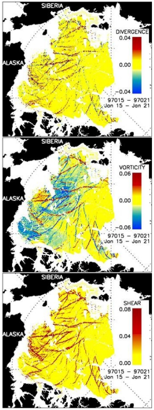
- Models do not accurately reproduce observed sea ice motion, drift, and deformation data (see *e.g.* Kwok *et al.* (2008) *JGR*, Girard *et al.* (2009) *JGR*, Rampal *et al.* (2011) *JGR*)



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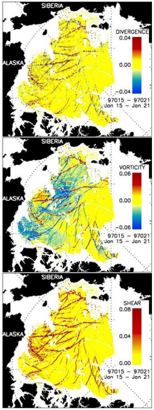

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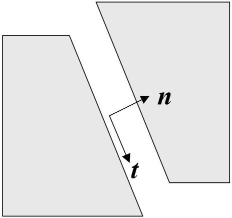
Want to develop improved rheologies to better approximate observed ice velocity and deformation.

Elastic-Decohesive Rheology

- Schreyer*et al.* (2006) *JGR*
- Leads modeled as displacement discontinuities $[\![\mathbf{u}]\!]$
- Intact ice modeled as elastic
- Predicts initiation and orientation of leads
- Once failure begins behavior is anisotropic



$$[\![\mathbf{u}]\!] = u_n\mathbf{n} + u_t\mathbf{t}$$

Yield Function

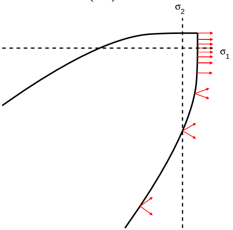
Elastic-Decohesive Rheology

$$\Phi(\boldsymbol{\sigma}) = \left(\frac{\tau_t}{\tau_{sm}}\right)^2 + \mathrm{e}^{\kappa B_n} - 1$$

$$B_n = \frac{\tau_n}{\tau_{nf}} - f_n \left(1 - \frac{\langle -\sigma_{tt} \rangle^2}{f_c'^2}\right)$$

$\boldsymbol{\sigma}$ = stress, $\boldsymbol{\tau} = \boldsymbol{\sigma} \cdot \mathbf{n}$ = traction

$$\Phi(\boldsymbol{\sigma}) = 0$$



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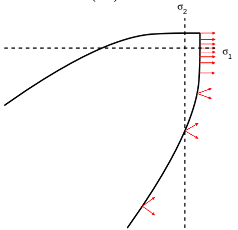

- τ_t tangential traction, τ_{sm} shear strength

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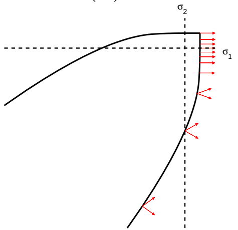

- τ_t tangential traction, τ_{sm} shear strength
- τ_n normal traction, τ_{nf} tensile strength

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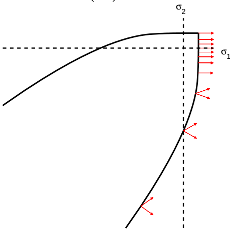

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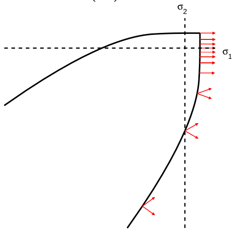
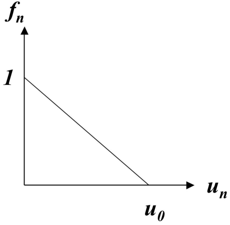
- τ_n normal traction, τ_{nf} tensile strength
- $\sigma_{tt} = \mathbf{t} \cdot \boldsymbol{\sigma} \cdot \mathbf{t}$, f_c' compressive strength
- Softening function $f_n = \langle 1 - u_n/u_0 \rangle$

$$\Phi(\boldsymbol{\sigma}) = 0$$





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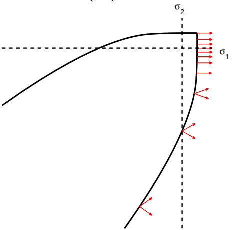
Viscous-Plastic Rheology

$$\Phi(\mathbf{N}) = \left(\frac{N_1 + N_2 + P}{P}\right)^2 + \left(\frac{N_2 - N_1}{P} e\right)^2 - 1$$

$\mathbf{N}$ = depth integrated stress

$\Phi(\boldsymbol{\sigma}) = 0$



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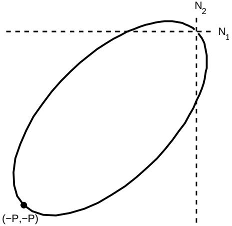


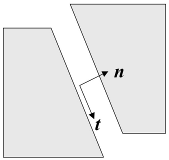
Constitutive Relations

Elastic-Decohesive Rheology

$$\dot{\boldsymbol{\sigma}} = \mathbb{E} : (\dot{\boldsymbol{\varepsilon}} - \dot{\boldsymbol{\varepsilon}}^d)$$

$$\dot{\boldsymbol{\varepsilon}}^d = \frac{1}{L}([\![\dot{\mathbf{u}}]\!] \otimes \mathbf{n})^s, \quad [\![\dot{\mathbf{u}}]\!] = \lambda \frac{\partial \Phi}{\partial \boldsymbol{\tau}}$$

Schreyer *et al. (2006)*



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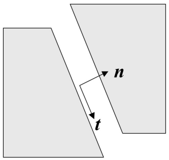

$$\llbracket \mathbf{u} \rrbracket = u_n \mathbf{n} + u_t \mathbf{t}$$

Viscous-Plastic Rheology

$$\mathbf{N} = 2\eta\dot{\boldsymbol{\varepsilon}} + (\zeta - \eta)\mathrm{tr}(\dot{\boldsymbol{\varepsilon}})\mathbf{I} + \frac{P\mathbf{I}}{2}$$

$$\zeta = \frac{P}{2\Delta}, \quad \eta = \frac{\zeta}{e^2} = \frac{P}{2\Delta e^2}$$

$$\Delta = \left((\dot{\varepsilon}_{11}^2 + \dot{\varepsilon}_{22}^2)(1 + e^{-2}) + 4\dot{\varepsilon}_{12}^2 e^{-2} + 2\dot{\varepsilon}_{11}\dot{\varepsilon}_{22}(1 + e^{-2})\right)^{1/2}$$

Hibler (1979)

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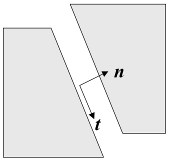

$$[\![\mathbf{u}]\!] = u_n \mathbf{n} + u_t \mathbf{t}$$

Elastic-Viscous-Plastic Rheology

$$\frac{1}{E}\frac{\partial \mathbf{N}}{\partial t} + \frac{1}{2\eta}\mathbf{N} + \frac{\eta - \zeta}{4\eta\zeta}\mathrm{tr}(\mathbf{N})\mathbf{I} + \frac{P}{4\zeta}\mathbf{I} = \dot{\boldsymbol{\varepsilon}}$$

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Hunke and Dukowicz (1997)

EDC Implementation in CICE

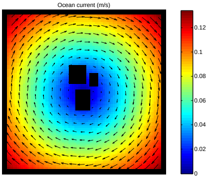
- Replace EVP depth-integrated stress with EDC depth-integrated stress ($N = h\boldsymbol{\sigma}$)
- Momentum equation solve is unchanged
- Open water fraction tied loosely to crack opening - both depend on velocity divergence

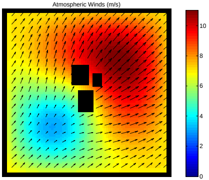
Algorithm

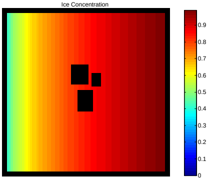
- Compute strain increment from velocity $\Delta\varepsilon = 1/2(\nabla\mathbf{v} + (\nabla\mathbf{v})^T)\Delta t$
- Compute trial stress from strain increment $\Delta\sigma^{tr} = \mathbb{E} : \Delta\varepsilon$
- Find new failure direction, if necessary $\max_n \Phi_n$
- Evaluate failure function $\Phi(\boldsymbol{\sigma})$
- If $\Phi(\boldsymbol{\sigma}) < 0$ step is elastic and $\Delta\sigma = \Delta\sigma^{tr}$
- Else find ε^d such that $\Phi(\boldsymbol{\sigma}) = 0$ using Newton's method
- Compute updated stress $\Delta\sigma = \mathbb{E} : (\Delta\varepsilon - \Delta\varepsilon^d)$

Box Test Problem

- Hunke (2001) *JCP*
- Rectilinear box grid, 80×80, $\Delta x = \Delta y = 16$ km
- Constant ice thickness of 2 m
- Ice concentration function of horizontal coordinate
- Ocean current constant in time







$$u_{ocn} = 0.1(2y - L_y)/L_y$$
$$v_{ocn} = -0.1(2x - L_x)/L_x$$

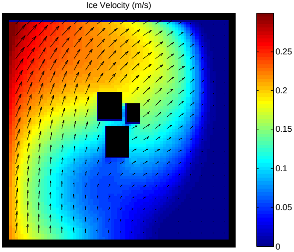
- Atmospheric winds, $T = 4$ days

$$u_{atm} = 5 + \left(\sin\left(\frac{2\pi t}{T}\right) - 3\right) \sin\left(\frac{2\pi x}{L_x}\right) \sin\left(\frac{\pi y}{L_y}\right)$$
$$v_{atm} = 5 + \left(\sin\left(\frac{2\pi t}{T}\right) - 3\right) \sin\left(\frac{2\pi y}{L_y}\right) \sin\left(\frac{\pi x}{L_x}\right)$$

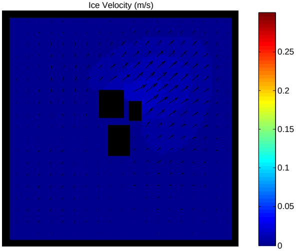
Box Test Problem

Ice Velocity after 3 Days

EVP



EDC



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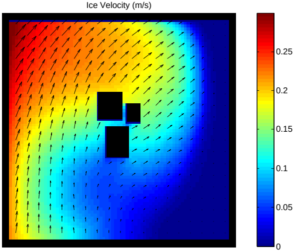

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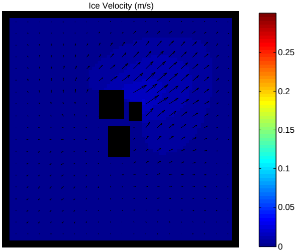
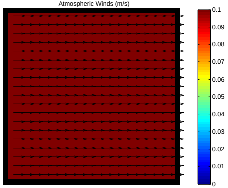


Initial concentration inconsistent with no initial cracks in EDC!

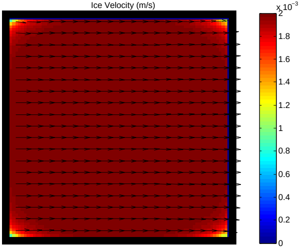
Symmetric Box Test

- Rectilinear box grid, 80×80, $\Delta x = \Delta y = 16$ km
- Constant ice thickness of 2 m
- Constant ice concentration of 0.5
- Zero ocean current
- Atmospheric winds constant in time, $u_{atm} = 0.1$ m/s, $v_{atm} = 0$

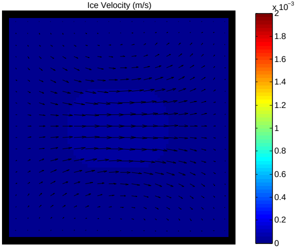
Wind



CICE EVP



CICE EDC



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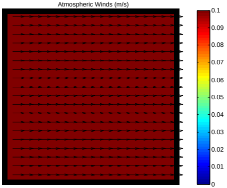

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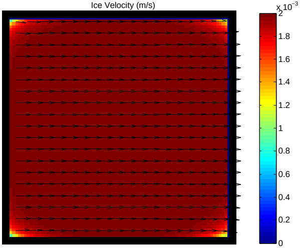

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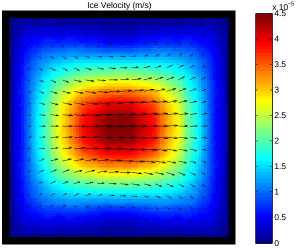
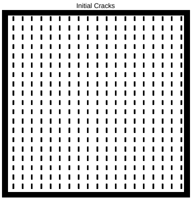
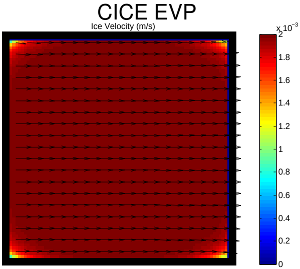


Uninitialized EDC runs are purely elastic

Symmetric Box Test with Initialization

- Set crack normal perpendicular to wind forcing
- Set normal opening based on ice concentration





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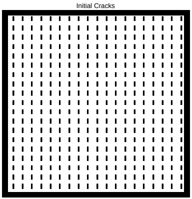

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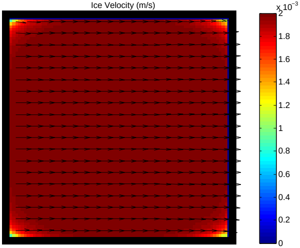

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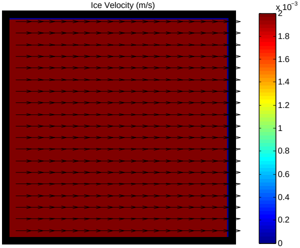
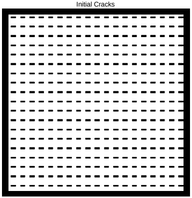
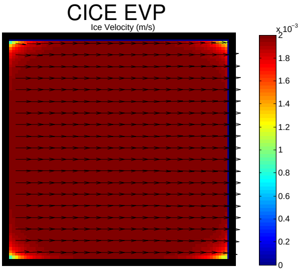
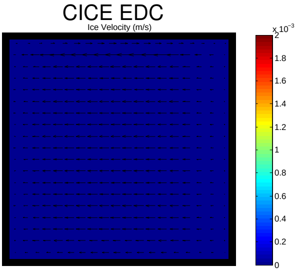


Initialized EDC results are consistent with EVP results

- Ice retains strength in unweakened directions
- Set crack normal parallel to wind forcing
- Set normal opening based on ice concentration







Initialization Algorithm

Given initial ice velocity:

- Compute strain rate at cell center

$$\dot{\boldsymbol{\varepsilon}} = \frac{1}{2}\left(\nabla\mathbf{v} + (\nabla\mathbf{v})^T\right)$$

- Compute stress increment from strain rate

$$\Delta\boldsymbol{\sigma} = \mathbb{E} : \dot{\boldsymbol{\varepsilon}} dt$$

- Maximize yield function to find direction

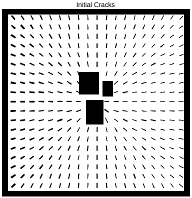
$$max_n \Phi(\Delta\boldsymbol{\sigma})$$

- Use concentration and cell dimensions to define crack width

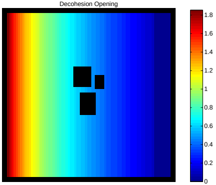
$$u_n = (1 - a_{ice})A_{cell}/L$$

Box Test Problem with Initialization

Initial Cracks



Initial Decohesion Opening



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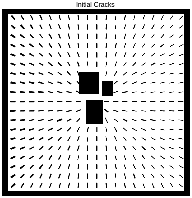

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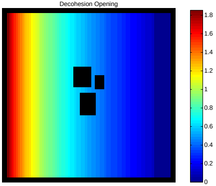
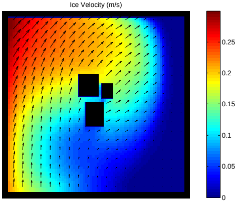
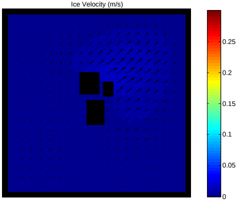


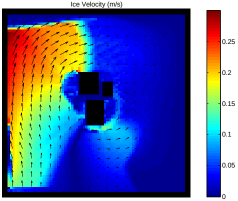
EVP



Uninitialized EDC



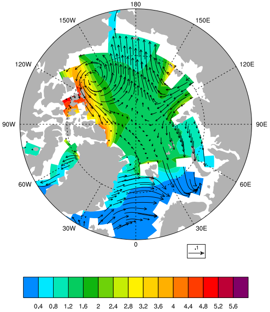
Initialized EDC



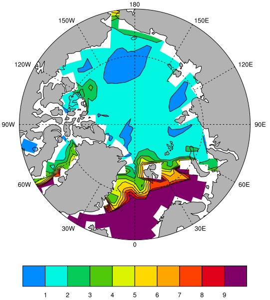
Simple Global Simulation

- Global 3 degree grid, 1997 data files
- Velocity, concentration, thickness initialized from EVP run
- Use initial ice velocity to predict crack direction

Initial Velocity and Thickness



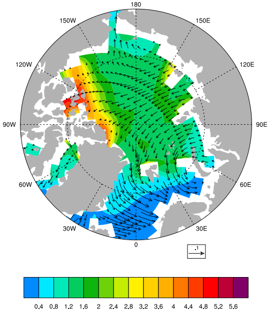
Initial Decohesion Opening



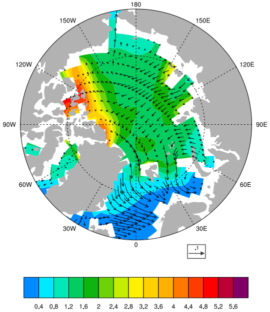
Simple Global Simulation

- Global 3 degree grid, 1997 data files
- Velocity, concentration, thickness initialized from EVP run
- Use initial ice velocity to predict crack direction

EVP Velocity and Thickness
after One Month



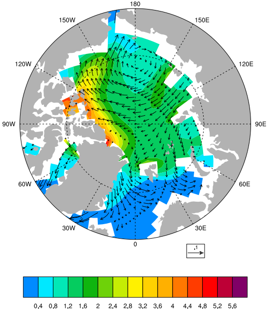
EDC Velocity and Thickness
after One Month



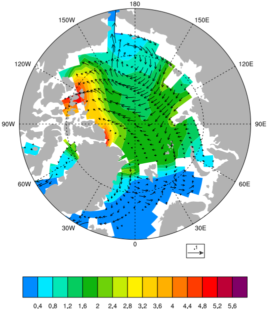
Simple Global Simulation

- Global 3 degree grid, 1997 data files
- Velocity, concentration, thickness initialized from EVP run
- Use initial ice velocity to predict crack direction

EVP Velocity and Thickness after One Year



EDC Velocity and Thickness after One Year



- Initial implementation of elastic-decohesive rheology complete
    - multiple cracks per cell
    - cracks initialized based on ice velocity and concentration
- Work to be done
    - model tuning and more testing
    - crack healing due to refreezing
    - advection of cracks
    - detailed comparison with RGPS deformation data